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Journal of Monetary Economics 53 (2006) 1509–1539
Journal of MONETARY ECONOMICS
www.elsevier.com/locate/jme

Durables, nondurables, down payments and consumption excesses ☆

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Received 16 February 2005; accepted 19 May 2005
Available online 5 June 2006

Abstract

We examine a model that generalizes the standard buffer-stock model of saving to accommodate durables, nondurables, down payment requirements, and adjustment costs in the durables market. We find that nondurable consumption becomes more volatile relative to income as down payments decrease at the individual and at the aggregate level. Moreover, for plausible parameter values, the model can explain the excess smoothness and excess sensitivity observed in U.S. aggregate data. The result follows from a gradual adjustment of consumption to permanent income shocks when agents attempt to spread out the burden of down payments over time, compounded by slow adjustment due to transaction costs.

JEL classification: E21; C36; C61

Keywords: Buffer stock; Consumption; Durable goods; Incomplete markets; Computational economics

☆ The author thanks John Driscoll, Darin Lee and an anonymous referee for very helpful comments. This paper is based on earlier joint work with Chris Farr and the author is particularly grateful to him for his support. I benefited from the feedback of seminar participants at Alicante, Brown University, Carlos III, CBO, Florida International, ITAM, Pompeu Fabra, and conference participants at the 16th Annual Congress of the European Economic Association and the 2001 North American Summer Meeting of the Econometric Society. Financial support from the Bank of Spain and from DGICYT, project BEC2000-0173, is gratefully acknowledged.

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doi:10.1016/j.jmoneco.2005.05.010

1. Introduction

Private consumption is the most important component of aggregate demand. Nevertheless, modelling consumption behavior is still a challenge to the economics profession. One of the lasting contributions to modern day consumer theory is the idea that consumption is determined by the expected value of lifetime resources or permanent income, known as the life cycle-permanent income hypothesis model (LC-PIH).[1] The modern-day specification of this fundamental concept involves an inter-temporal choice model with quadratic preferences, stochastic labor income, no borrowing restrictions, and perfect foresight. In spite of its intuitive appeal, several influential papers revealed discrepancies between this model's predictions and the aggregate data. Hall (1978) surprised the profession by demonstrating that under simple assumptions, consumption is a martingale; i.e., a regression of period t consumption growth on any variable known at period $t-1$ should return an estimate of zero. Regressions using aggregate data, however, consistently return an estimate significantly larger than zero when current growth in consumption is regressed on lagged aggregate income growth—a phenomenon known as 'excess sensitivity' of current consumption to lagged income.[2] Likewise, the LC-PIH model predicts that if the income process exhibits high persistence, current consumption should respond strongly to unanticipated innovations. Yet, empirical work using aggregate data consistently finds a small reaction of consumption to current income shocks—a phenomenon known as 'excess smoothness'.[3]

The buffer-stock model of saving, pioneered by Deaton (1991) and Carroll (1997), is a promising candidate for replacing the LC-PIH model as the benchmark model of consumer behavior. This model, while still preserving many of the insights stemming from rational forward-looking behavior, assumes individuals cannot (or endogenously will not) borrow and allows consumers to be more prudent and less patient than in Hall (1978). Over the last decade, a large body of literature has shown that the buffer-stock model can explain several aspects of household spending decisions. However, at the aggregate level, the implications of the buffer-stock model are not as well explored, and in some cases, not fully satisfactory. Ludvigson and Michaelides (2001) showed—in a careful and explicit aggregation of the buffer-stock model—that the model cannot generate robust excesses. The authors rely on incomplete information as in Pischke (1995) to generate some excesses.

We believe one main shortcoming of most consumption models, including the standard buffer-stock model, is that they traditionally focus solely on the study of nondurable consumption.[4] In this paper, we present a generalized buffer-stock model with durables and nondurables that can explain the excess sensitivity and excess smoothness of nondurable consumption observed in the aggregate data without the need for incomplete information.

Introducing durable goods into this framework is not straightforward and can be done in several different ways. In our specification, we assume that individuals derive utility

[1]This idea was pioneered by Modigliani and Brumberg (1954) and Friedman (1957).

[2]For studies of excess sensitivity, see Flavin (1981) and Campbell and Deaton (1989).

[3]Campbell and Mankiw (1987) and Cochrane (1988) showed that innovations to GNP are highly persistent. Building on the results in Hansen and Sargent (1981), excess smoothness was documented in Campbell and Deaton (1989).

[4]Notable exceptions are Caballero (1993) and Eberly (1994) who focus on durables, and Chah et al. (1995), Alessie et al. (1997) and Carroll and Dunn (1997), who consider models with both durables and nodurables.

from consumption of a nondurable good and from the services provided by a durable good. Moreover, the durable good serves as collateral for credit purchases. In particular, the durable (minus a down payment) can be financed, and durable-equity loans (with a certain maximum loan-to-value ratio) are available to consumers. Collateralized constraints of this type impose distortions on the allocation of consumption across time and across goods, even with a utility function separable in both goods.[5] We also take into account the fact that the market for durables may be characterized by important transaction costs. We consider nonconvex costs of adjustment as in Grossman and Laroque (1990), which generate large and infrequent adjustments in a (S, s) rule fashion. In all other respects, our model is identical to the classic buffer-stock framework.

The use of a collateralized constraint should not be controversial. First, according to the Federal Reserve Board's 1998 *Survey of Consumer Finances* (SCF), collateral borrowing, mainly obtained to purchase housing and automobiles, is the principal type of borrowing undertaken by households. For example, in the 1998 SCF, 92% of all available credit to households was for the purchase of houses and automobiles (collateral credit). Moreover, the average ratio of collateral credit to total debt across households was roughly 79%. Second, considering this constraint allows us to study an extra motive for saving—burdensome down payments—and to analyze certain implications of financial liberalization. According to the annual survey of home buyers *Who's Buying Homes in America?*, households must save, on average, for two and a half years to buy their first home.[6] Moreover, the Federal Housing Finance Board reports that the average down payment for single-family dwellings went down from 26% in the 1970s to 22% in the 1990s. Likewise, the percentage of homeowners putting down less than 10% went up from 12% in the 1970s to 19.8% in the 1990s. The proliferation of home-equity loans has also been dramatic.

It is important to acknowledge that a similar formulation of the problem *without* adjustment costs has been explored by Chah et al. (1995) and Alessie et al. (1997). However, the focus of these papers is empirical. Carroll and Dunn (1997) present a similar model with adjustment costs but concentrate on the role of unemployment expectations on consumption spending decisions. To our knowledge, this particular version of the model has not yet been explored. We show that with a combination of the right numerical dynamic programming techniques, the *curse of dimensionality* and the complications that the illiquidity of the durable poses can be overcome, and reasonable parameterizations of the model can be solved accurately. In particular, we use Euler equation iteration for a version of the model with no adjustment costs in the durable market, and a finite state approximation method for the version with adjustment costs.

After solving the model, we characterize the optimal consumption rules for an individual consumer under different down payment regimes. We then simulate individual and aggregate consumption series—calculated through explicit aggregation—and study the implications of changing down payments on consumption patterns. Finally, we explore whether a plausible parametrization of the model can account for the excess sensitivity and excess smoothness observed in the aggregate data for nondurable consumption.

[5] Even without collateralized liquidity constraints, the interactions between durable and nondurable goods are interesting. Browning and Crossley (1997) show that individuals who face limited borrowing alternatives smooth out fluctuations in income by postponing the replacement of small durables.

[6] See Chicago Title and Trust, 1995. For first-time buyers, the majority of the required down payment comes from savings (74.8%). For repeat buyers, savings are supplemented by the proceeds from the existing house sale (52.2%from savings and 31.7% from the sale of the previous home).

At the individual level, we find that nondurable consumption becomes smoother relative to income as down payment requirements increase for two different reasons. First, when income is transitorily low, a buffer-stock consumer on occasion liquidates the equity accumulated in the durable to prop up his nondurable consumption. Since higher required down payments translate into higher levels of equity, nondurable consumption becomes smoother. Second, when an individual experiences a positive permanent income shock, he chooses not to fully adjust his consumption due to the desire to spread out the cost of accumulating the down payment. The implications for the durable are slightly more complex and are thoroughly discussed throughout the paper.

At the aggregate level, nondurable consumption is also smoother for higher down payments. The result follows from the gradual adjustment of consumption due to the consumer's desire to spread out the cost of the down payment. At this level, all individual-specific shocks cancel out and what remains is the effect of aggregate shocks, which we model as permanent. Furthermore, the sluggish response of consumption to changes in income can generate robust excesses for reasonable parameter values, especially in the presence of transaction costs.

In brief, we find that with lower down payments, aggregate consumption becomes more volatile relative to income and excess sensitivity weakens.[7] A further interesting implication of the model is that average wealth holdings fall with decreases in down payments or transaction costs. These results are consistent with a variety of findings in the empirical macroeconomic literature. Japelli and Pagano (1989) find that down payment requirements are highly correlated with excess sensitivity. Bacchetta and Gerlach (1997) find that excess sensitivity varies over time with a clear tendency to decline in U.S. aggregate data. More recently, Peersman and Pozzi (2004) show an inverse relation between the excess sensitivity coefficient and measures of financial liberalization in the U.S. Likewise, Kose et al. (2003), using international macroeconomic data, document that the volatility of consumption relative to income increases with the degree of financial integration.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 describes the solution method, the calibration, as well as the optimal policy functions for nondurable and durable consumption. In Section 4, we discuss the implications of the model on consumption patterns for both an individual household and for the aggregate. Section 5 provides concluding remarks.

2. The basic model

A consumer maximizes the present discounted value of expected utility from consumption of a nondurable good, C_t, and from the service flow provided by a durable good, K_t, where t denotes time. We assume that time is discrete and agents face an infinite

[7]This prediction is not inconsistent with the evidence that consumption volatility and output volatility decrease with financial development as documented for example in Denizer et al. (2002). Our model predicts increases in the volatility of consumption *relative* to the volatility of income. If financial liberalization reduces the volatility of income, the absolute volatility of consumption would decrease in our model as well.

horizon. $\beta < 1$ is the discount factor.[8]

$$\max_{\{C_t, K_t\}} V = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, K_t) \right\}. \tag{1}$$

The instantaneous utility function is assumed to be separable in both goods and is of the CRRA type:

$$U(C_t, K_t) = \frac{C_t^{1-\rho}}{1-\rho} + \varphi \frac{K_t^{1-\rho}}{1-\rho}, \tag{2}$$

where φ is a preference parameter.[9] $\rho > 0$ implies that the agent is risk averse and has a precautionary motive for saving.

This maximization is subject to both a budget constraint and a wealth constraint. Assume there is one riskless financial asset, A_t. R is the interest factor paid on it. In period t, an agent holds past financial assets gross of interest, RA_{t-1}, and receives Y_t units of income. In the same period, the agent chooses nondurable consumption, C_t, and net investment on the durable, $(K_t - \psi K_{t-1})$, where ψ is the depreciation factor. Also, the agent may be subject to an adjustment cost, $\zeta(K_t, K_{t-1})$, when changing the durable stock. Thus, we can write the budget constraint between two successive periods as follows:

$$A_t = RA_{t-1} + Y_t - C_t - (K_t - \psi K_{t-1}) - \zeta(K_t, K_{t-1}). \tag{3}$$

Labor income is assumed to be exogenous to the agent and stochastic. It is the only source of uncertainty in the model. We assume, as in Ludvigson and Michaelides (2001) and similar to Carroll (1997), that labor income, Y_t, is the product of permanent income, P_t, and an idiosyncratic transitory shock, T_t: $Y_t = P_t T_t$. In turn, permanent income is $P_t = G_t P_{t-1} N_t$. G_t can be thought of as the growth in permanent income attributable to aggregate productivity growth in the economy which is common to all agents. N_t is a permanent idiosyncratic shock. We assume $\ln G_t$, $\ln T_t$, and $\ln N_t$ are independent and identically normally distributed with means μ_G, μ_T, and μ_N, and variances σ_G^2, σ_T^2, and σ_N^2, respectively. This income specification is particularly useful since it allows for consumers to share in general growth while the variance of their income can be calibrated to be dominated by idiosyncratic permanent or transitory components. The specification implies that the growth rate of individual labor income follows an MA(1) process, $\Delta \ln Y_t = \ln G_t + \ln N_t + \ln T_t - \ln T_{t-1}$, which is consistent with the microeconomic evidence.[10] By the law of large numbers, aggregate income, $\bar{Y}_t$, follows the process, $\Delta \ln \bar{Y}_t = \ln G_t + 0.5\sigma_N^2$, where $\bar{Y}_t = 1/n \sum_{i=1}^{n} Y_{it}$.

[8]Note the simplifying assumption that an agent's service flow from the durable is proportional to the durable stock, with the constant of proportionality equal to one. A slightly more realistic setup would express utility as a function of the service flows derived from the durable stock. These services could be affected by, among other things, frequency of use.

[9]We follow Bernanke (1984) who studies the joint behavior of the consumption of durable and nondurable goods and finds that separability was a good approximation. With regards to prices, we assume that $P_t^C / P_t^K = 1$, $\forall t$.

[10]Empirical studies such as MaCurdy (1982), Abowd and Card (1989), and Pischke (1995) find that an MA(1) in the growth rate of income is a good approximation to models estimated on microeconomic data.

A very important aspect of the model is the collateralized constraint imposed on the agent:

$$A_t + (1-\theta)K_t \geqslant 0, \tag{4}$$

with $\theta \in [0,1]$. This constraint implies that an individual's borrowing limit is a fraction $(1-\theta)$ of the durable stock. The constraint summarizes several commonly observed aspects of collateral lending. A household can only finance a fraction $(1-\theta)$ of durable purchases. In other words, it must satisfy a down payment requirement θ. The constraint also implies that when a household owns a durable good, it can obtain a durable-equity loan with a maximum loan-to-value ratio $(1-\theta)$. In summary, at any point in time, an agent is only required to keep an accumulated durable equity equal to θK_t. Note that total wealth, $A_t + K_t$, can be divided into a *required down payment* (required equity or required wealth), θK_t, and the wealth held in excess of the required down payment or *voluntary equity*, $Q_t \equiv A_t + (1-\theta)K_t$. Also, the consumer can increase nondurable consumption by decreasing either voluntary or required equity. However, accessing required equity implies changing the durable stock, which may be costly if adjustment costs are present. Finally, this collateralized constraint does not imply a fixed borrowing limit but a limit that varies with the durable stock and θ.

3. Solving the model

A closed-form solution of the model does not exist and we must rely on computational methods to solve the consumer's problem. This section presents our computational strategy as well as some qualitative implications of the solution. We first solve the model with no adjustment costs to understand the role of durability and the collateralized constraint. We then add nonconvex adjustment costs to incorporate irreversibility and infrequent changes in durable purchases.

3.1. No adjustment costs

Euler equation iteration has been the traditional approach for solving microeconomic dynamic stochastic optimization problems with nondurable consumption only. We generalize the algorithm in Carroll (1997) and Deaton (1991) to accommodate multiple goods and the collateralized constraint considered here. We reformulate the model to facilitate the implementation of this numerical technique.

3.1.1. Reformulating the model

Define cash-on-hand, X_t, as $X_t \equiv RA_{t-1} + \psi K_{t-1} + Y_t$. Note that durable wealth can be lumped together with financial resources and labor income because we assume for now that there are no costs of adjusting the durable stock. The budget constraint becomes $A_t = X_t - C_t - K_t$ and the collateralized constraint $C_t + \theta K_t \leqslant X_t$. Combining the definition of cash-on-hand and the budget constraint, we can write an expression for the evolution of cash-on-hand: $X_{t+1} = R(X_t - C_t) + (\psi - R)K_t + Y_{t+1}$. The first-order conditions of the problem are:

$$U_C^t = \beta R \mathrm{E}_t[U_C^{t+1}] + \lambda_t, \tag{5}$$

$$U_K^t = \beta(R-\psi)\mathrm{E}_t[U_C^{t+1}] + \theta\lambda_t, \tag{6}$$

$$\lambda_t(X_t - C_t - \theta K_t) = 0. \tag{7}$$

Eq. (5) states that the marginal utility of nondurable consumption in period t must equal the discounted expected marginal utility of nondurable consumption in period $t+1$, plus the shadow price of the constraint. Analogously, Eq. (6) states that the marginal utility of durable consumption in period t must be equal to the expected marginal utility of nondurable consumption in period $t+1$ discounted by $\beta(R-\psi)$, plus θ times the shadow price of the constraint. Note the difference in the discount factor—βR versus $\beta(R-\psi)$—because of the durability of K. Also, λ_t is multiplied by θ in Eq. (6) to reflect the fact that only a down payment is required as payment for the durable in period t.

Eqs. (5) and (6) are inter-temporal conditions. Solving for $\beta\mathrm{E}_t[U_C^{t+1}]$ in Eq. (5) and plugging it into Eq. (6), we obtain an equation for the intra-temporal relationship between C_t and K_t:

$$U_K^t = \frac{R-\psi}{R}U_C^t + \left(\theta - \frac{R-\psi}{R}\right)\lambda_t. \tag{8}$$

Note that when the liquidity constraint is not binding, $\lambda_t = 0$. Thus, given our utility function, Eq. (8) implies

$$\frac{C_t}{K_t} = \varphi^{-1/\rho}\left(\frac{R-\psi}{R}\right)^{1/\rho} \equiv \Omega, \tag{9}$$

which is the optimal relationship between C_t and K_t accounting for durability.[11] $(R-\psi)/R$ is known in the literature as the user cost of the durable: the single-period cost, or rental equivalent cost of one durable unit. The user cost depends on both the depreciation factor and the interest rate. Depreciation erodes the agent's investment in the durable and effectively increases its cost. The interest rate also increases the user cost as it reflects the opportunity cost of investing in the durable: a dollar invested in the durable could have returned $R-1$ dollars if invested in financial assets.[12] When the agent is not constrained, the trade-off between C_t and K_t is fully captured by the user cost and the preference parameters. For constrained agents, other factors come into play. If K_t is poor collateral (θ is higher than the user cost), constrained agents let durable consumption fall temporarily and vice versa. Note that when $\theta = (R-\psi)/R$, the trade-off between C_t and K_t is determined only by the user cost, even if the constraint is binding. This is a particularly useful benchmark case since the constraint does not impose any distortions in the intra-temporal allocation between the two goods.

In order to deal with the nonstationarity of income, we normalize all variables by permanent income, P_t, as proposed by Carroll (1997). Lower-case variables denote upper-case counterparts divided by permanent income. The Euler–Lagrange equations can be

[11] If $\varphi = 1$ and ψ is 0 (i.e., the durable depreciates completely after one period), $U_C^t = U_K^t$. That is, the agent would choose to consume the same amounts of both goods ($C_t = K_t$). If $\psi > 0$, $C_t < K_t$.

[12] Note that we ignore other factors, such as capital gains and losses on the durable. A more general specification for the user cost would be $(P_t^K R - P_{t+1}^K\psi)/(P_t^K R)$.

rewritten as follows. If the agent is not constrained:

$$\beta R \mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left(c_{t+1} \left[(G_{t+1}N_{t+1})^{-1} \left(R(x_t - c_t) + \left(\frac{\psi - R}{\Omega} \right) c_t \right) + T_{t+1} \right] \right)^{-\rho} \right\} - c_t^{-\rho} = 0. \tag{10}$$

If the agent is constrained:

$$\beta[\psi - R(1-\theta)]\mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left(c_{t+1} \left[(G_{t+1}N_{t+1})^{-1} [\psi - R(1-\theta)] \frac{x_t - c_t}{\theta} + T_{t+1} \right] \right)^{-\rho} \right\} - \theta c_t^{-\rho} + \varphi \left(\frac{x_t - c_t}{\theta} \right)^{-\rho} = 0. \tag{11}$$

Eqs. (10) and (11) can be solved to obtain a policy function for normalized nondurable consumption as a function of the only state variable, normalized cash-on-hand, $c(x)$. Once we find the policy function for nondurable consumption, the policy function for durable consumption, $k(x)$, can be calculated by using the intra-temporal relationship between the two goods. Appendix A.1 presents further details, including convergence conditions.

3.1.2. The policy functions

We now describe the shape of the optimal consumption functions for normalized nondurable and durable consumption. In order to compare our findings with the previous literature, we start by adding c to θk. The policy function for this variable is depicted in Fig. 1A. Similar to Deaton (1991), there is a unique $x^*(\theta)$ such that:

$$c + \theta k = \begin{cases} x, & x \leqslant x^*(\theta), \\ < x, & x > x^*(\theta). \end{cases}$$

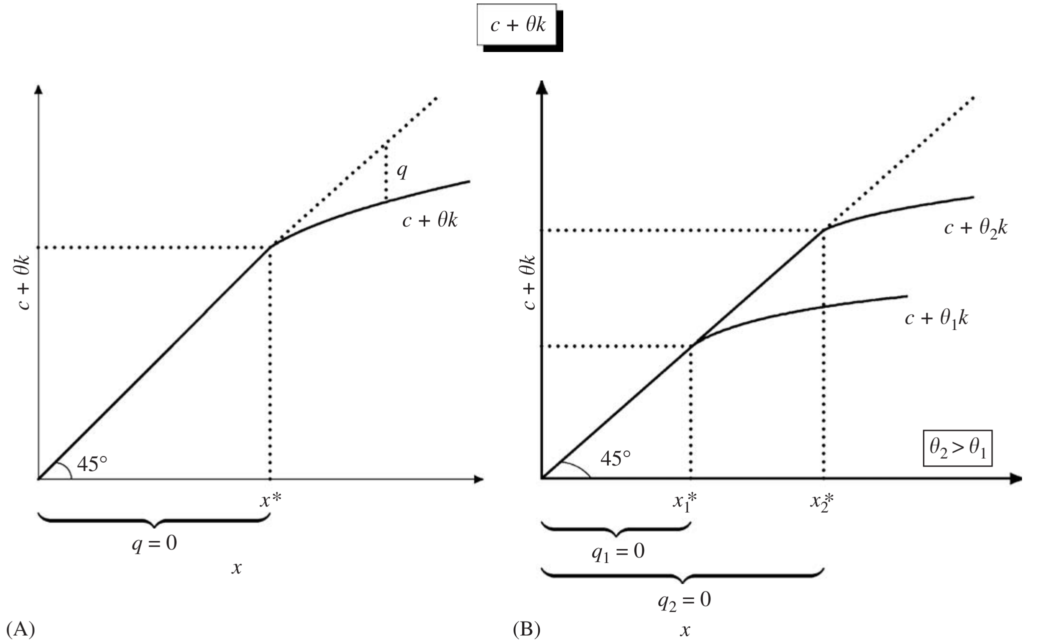

Fig. 1. Policy functions. Nondurable plus down payment as a function of cash-on-hand.

If normalized cash-on-hand, x, is below a threshold level, $x^*(\theta)$, the agent is constrained and all resources are exhausted after paying for nondurable consumption and the down payment requirement. In other words, no normalized voluntary equity is carried over to the next period, $q \equiv a + (1-\theta)k = 0$. If cash-on-hand is higher than $x^*(\theta)$, some voluntary equity is accumulated, $q > 0$. Note that the higher θ is, the higher the required level of equity an agent must keep. As a result, the threshold level $x^*(\theta)$ is an increasing function of θ (see Fig. 1B).

How are resources are allocated between the two goods? Propositions 1 and 2 summarize our findings. Proofs are presented in Appendix B.

Proposition 1. *When an agent is not constrained,* $x > x^*(\theta)$, $c(x) = \Omega/(\Omega+\theta)(x-q)$, *and* $k(x) = (1/(\Omega+\theta))(x-q)$, *regardless of the value of* θ.

In words, when the agent is not constrained and once he has made the decision regarding how much voluntary equity to bring to the next period, the agent spends fixed proportions of the remaining cash-on-hand between the two goods.

Proposition 2. *When an agent is constrained,* $x \leqslant x^*(\theta)$,

(A) *If no down payment is required,* $\theta = 0$,

$$c(x) = x \quad \text{and} \quad k(x) = \frac{1}{\Omega} x^*(\theta).$$

(B) *If the down payment parameter is lower than the user cost,* $\theta < (R-\psi)/R$, $c(x)$ *is a convex function of* x, *and* $k(x)$ *is a concave function of* x.

(C) *If the down payment parameter is equal to the user cost,* $\theta = (R-\psi)/R$, $c(x)$ *and* $k(x)$ *are linear functions of* x. *In particular*:

$$c(x) = \frac{\Omega}{\Omega+\theta} x \quad \text{and} \quad k(x) = \frac{1}{\Omega+\theta} x.$$

(D) *If the down payment parameter is higher than the user cost,* $(R-\psi)/R < \theta \leqslant 1$, $c(x)$ *is a concave function of* x, *and* $k(x)$ *is a convex function of* x.

Proposition 2 describes, for the case when the agent is constrained, the shapes of the policy functions which depend on the credit conditions characterized by the relationship between the down payment parameter and the user cost. Fig. 2 illustrates the propositions by depicting the policy rules for the four parameter regions described in Proposition 2.

For a constrained agent, the policy function for the durable good becomes 'flatter' as the down payment requirement gets lower. The opposite is true for nondurable consumption. This implies that for a level of normalized cash-on-hand such that the agent is constrained in all regimes, the marginal propensity to consume out of cash-on-hand for the nondurable is higher the lower the θ (in fact, it is exactly one when $\theta = 0$). The shapes suggest, therefore, higher nondurable volatility and lower durable volatility for regimes with lower down payments, but do not prove it. First, x is endogenous, and second, as θ gets higher, agents are more likely to be constrained for a given level of x. Whether or not nondurable (durable) consumption indeed becomes more (less) volatile relative to income with decreasing down payments needs to be verified through simulation.

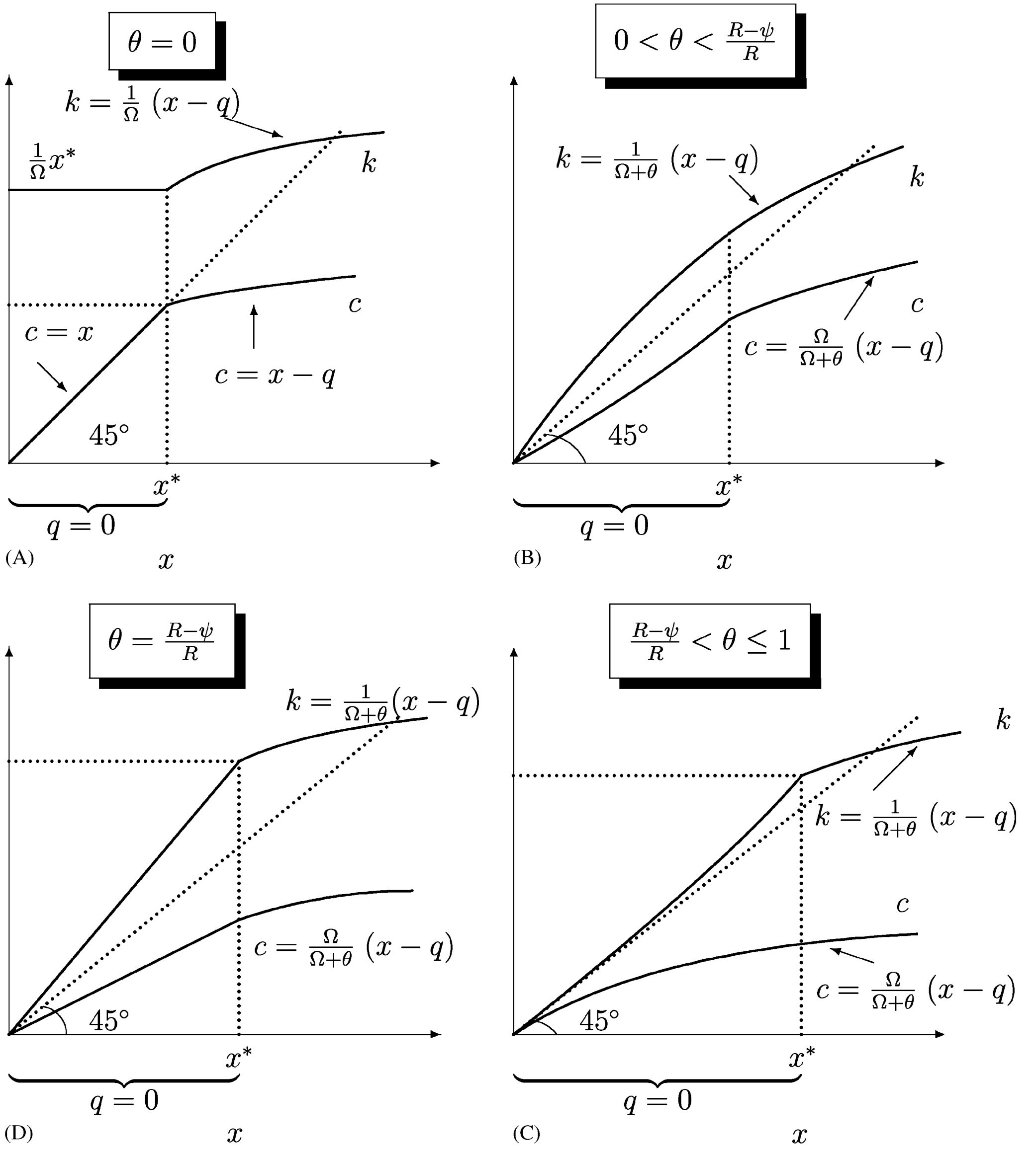

Fig. 2. Policy functions. Durable and nondurable as a function of cash-on-hand.

3.2. *Adjustment costs*

After gaining some understanding on the role of the collateralized constraint, we incorporate transaction costs in the durables market. Durables are typically purchased in large lumpy increments and changed only infrequently. Several authors argue that optimal consumption rules for durables can be described as following an (*S*,*s*) rule (e.g., Caballero, 1993; Eberly, 1994). When the stock of a durable good falls below some lower bound *s*, a purchase is made and the stock is readjusted to a target size S. As long as the stock of the durable good remains above the trigger point s, no action is taken. Nonconvex costs of adjustment generate (S,s) patterns and thus we choose a nonconvex cost specification in our model. In particular, we use a similar specification to Grossman and Laroque (1990):

$$\zeta(K_t, K_{t-1}) = d\phi\psi K_{t-1},$$

where ϕ is the adjustment cost parameter and d is a dummy variable which takes on the value of zero when there is no investment, $K_t - \psi K_{t-1} = 0$, and one otherwise. This adjustment cost can be seen as a proportional loss in the selling price of the agent's prior holdings of the durable stock. This loss in price can be attributable to any type of cost incurred upon sale, such as the payment of taxes, a sales commission, or an imperfection in the resale market for the durable.[13] Note that once the agent has decided to adjust his durable holdings, the adjustment cost is fixed from his perspective; the cost is proportional to the inherited level of the durable stock, ψK_{t-1}.[14] In this formulation, the transaction cost does not diminish in importance as households become wealthier, as with a purely fixed cost. The specification also implies that incremental adjustments do not occur (i.e., the agent must sell his entire existing stock upon adjustment).[15]

Technically, adding adjustment costs is not a trivial modification. We cannot use Euler equation iteration to solve the model because of nondifferentiability issues. Thus, we apply a different numerical dynamic programming technique, a finite state approximation method. In order to apply this technique, we must use an alternative reformulation of the model in terms of voluntary equity and the durable stock.

Given the adjustment cost specification, financial assets, A_t, evolve as follows:

$$A_t = RA_{t-1} - (K_t - \psi(1 - d\phi)K_{t-1}) + Y_t - C_t. \tag{12}$$

The evolution of voluntary equity is given by

$$\begin{aligned} Q_t &\equiv A_t + (1-\theta)K_t \\ &= RA_{t-1} - (K_t - \psi(1 - d\phi)K_{t-1}) + Y_t - C_t + (1-\theta)K_t \\ &\quad + R(1-\theta)K_{t-1} - R(1-\theta)K_{t-1} \\ &= RQ_{t-1} + [\psi(1 - d\phi) - R(1-\theta)]K_{t-1} - \theta K_t + Y_t - C_t. \end{aligned} \tag{13}$$

The constraint becomes $Q_t \geqslant 0, \forall t$. In order to deal with the nonstationarity of income, we normalize all variables by permanent income. Then, we use the homogeneity of degree

[13] See Lam (1989) for an analysis of the aggregate implications of the time series properties of durable expenditure when the irreversibility of incremental adjustment of the durable is due to resale market imperfections.

[14] In this specification the adjustment cost is paid by the seller. Alternatively, we could divide the cost between buyer and seller: $\zeta(K_t, K_{t-1}) = \phi_1 d\psi K_{t-1} + \phi_2 dK_t$. Then, the adjustment cost for a consumer when purchasing the durable would be $\theta + \phi_2$. In order to keep the effects of down payments separate from the effects of adjustment costs, we choose the first specification.

[15] This is more reasonable if K represents a single durable good (i.e., a house) than if K represents a basket of goods.

$(1-\rho)$ of the utility function to write the Bellman equation of the model as follows:

$$\begin{aligned}
&V(q_{t-1}, k_{t-1}) \\
&= \beta \mathrm{E}_{t-1}\bigg\{(G_t N_t)^{1-\rho} \max_{q_t,k_t;q_t \geqslant 0} U[(G_t N_t)^{-1}\{Rq_{t-1} + [\psi(1-d\phi) - R(1-\theta)]k_{t-1}\} \\
&\quad -\theta k_t + T_t - q_t, k_t] + V(q_t, k_t)\bigg\}. \qquad (14)
\end{aligned}$$

The solution technique consists of specifying and solving a finite-state problem that approximates the continuous one presented above (see Appendix A.2).

Adding adjustment costs obviously changes the policy functions for durables. Also, the role of durables as a substitute for a liquid buffer stock of saving is diminished since selling durables to recover required equity is costly. In order to fully understand the effect that down payment requirements and transaction costs have on consumption behavior, we solve both versions of the model numerically and calculate several consumption statistics for both nondurable and durable goods under different down payment regimes (different θs), with and without adjustment costs. A numerical solution requires appropriate calibration of the model's parameters, which we now describe.

3.3. Calibration

We use an annual horizon as most microeconomic evidence for the parameters that we must calibrate comes from studies of annual data. The relative risk aversion coefficient is $\rho = 2$. The discount rate and the net real interest rate are 5% and 2%, respectively ($\beta = 1/1.05$, $R = 1.02$). The income shocks parameters are: $\mu_G = 0.02$, $\mu_N = \mu_T = 0$, $\sigma_G = 0.025$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. All values are similar to Ludvigson and Michaelides (2001), whose results we compare to ours.[16] The adjustment cost parameter, ϕ, is 0 in the nonadjustment cost case and 5% in the adjustment cost case.

ψ, the depreciation factor, is set to 0.915, implying an annual depreciation rate of 8.5%. We obtain this number by combining data from the *National Income and Product Accounts* (NIPA) and the *Fixed Assets and Consumer Durable Goods Accounts* (FACD) from the Bureau of Economic Analysis for the years 1959–2001. We interpret durables, K, in a comprehensive manner as the sum of residential stocks and all consumer durable goods. Accordingly, investment on durables, I, is calculated as expenditure on consumer durables plus residential private domestic investment. We assume the U.S. is in a steady state and calculate the real, average ratio of investment on durables to the durable stock, which determines the depreciation rate: $1 - \psi = I/K$.

We need to calibrate one last parameter, φ, the preference parameter in the utility function. We proceed as follows. First, we find the ratio of real nondurable consumption to the durable stock (C/K) using NIPA and FACD, which is 0.36.[17] We know that when an

[16]In fact, these parameter values correspond to one of Ludvigson and Michaelides's (2001) worst performing scenarios. Results are robust to small variations of the parameters, which we do not tabulate here for brevity.

[17]C is defined as the sum of nondurable consumption plus services minus housing, and K as the sum of the private residential stock plus the stock of consumer durables. We keep shoes and clothing within the nondurable category. The results of the paper do not change significantly if these are ignored or treated as durables.

agent is not constrained:

$$\frac{C}{K} = \left(\frac{R-\psi}{R}\right)^{1/\rho} \varphi^{-1/\rho}.$$

Given the values of ρ, R, ψ and C/K, we obtain $\varphi = 0.795$. We use this number for our individual consumption simulations and let the C/K ratio vary accordingly to illustrate the effects of relevant parameters on the ratio. For the aggregate consumption simulations, φ is adjusted to keep the ratio C/K constant and equal to 0.36 under the different scenarios.

In our simulations, the down payment parameter, θ, is set free for two reasons. First, our durable good is a composite of very different commodities: houses, cars, furniture, etc. θ is the down payment parameter and $(1-\theta)$ is the maximum loan-to-value ratio for durable-equity loans. Not only are down payments likely to be different for the different categories, but while home-equity loans are widely available, other durable-equity loans at favorable rates are not as common. Second, certain aspects of the collateralized constraint we consider in this paper deviate from financial contracts written in reality. Mainly, in order to keep the model tractable, the down payment parameter is the same for all consumers and the borrowing rate is not a function of θ.[18] Therefore, it is not obvious what the right value for θ should be. Since for our sample period houses represent 82% of the total durable stock in FADC, we could argue then for values close to down payments for houses. According to the Federal Housing Finance Board, the average down payment for the period 1963–2001 is 25%. In fact, less than 33% of homeowners put down less than 20%. However, we anticipate that in order to quantitatively account for the aggregate excesses, we require a sufficient wedge between the user cost and the down payment requirement. Given our parameter choices, the user cost of the durable is roughly 10% for this calibration.[19]

We use these parameter values to compute individual optimal policy rules for normalized nondurable and durable consumption. Then, we generate labor income shocks from the assumed distribution of idiosyncratic and aggregate shocks for 200 periods. Given the optimal policy rules and the simulated income realizations, we calculate nondurable and durable consumption (for that number of periods) for several individuals. In order to explore both the microeconomic and macroeconomic implications of the model, we run two different sets of simulations. For the individual results, we compute relevant statistics for each individual time series (i.e., consumption growth, volatility of consumption, etc.), reporting the average of these statistics across several consumers. For the aggregate results, first, we calculate a time series of aggregate consumption and aggregate income as the average of individual consumption and income across consumers, and then compute the relevant statistics (i.e., we aggregate explicitly).

[18]Besides the collateral requirement, lenders impose several additional criteria to reduce the likelihood of default. For housing, some lenders require that the mortgage payment does not exceed some percentage of current income. Another standard condition requires the loan-to-value ratio to be below a certain threshold. Otherwise, the borrower faces higher marginal borrowing costs, including a higher interest rate and the purchase of mortgage insurance.

[19]In this paper, we abstract from house price appreciation which would reduce the user cost. If this possibility was introduced, the model may be able to account for the aggregate excesses for lower down payment parameters.

4. Implications of changing down payments

4.1. Individual consumption

Table 1 summarizes several microeconomic consumption statistics for different down payment regimes. We report average consumption growth, as well as excess sensitivity and excess smoothness coefficients for both nondurable and durable consumption. The excess smoothness coefficient is calculated as the ratio of the standard deviation of consumption growth to that of income growth. It is therefore a measure of the relative volatility of consumption. The excess sensitivity figure is the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. It is one of the possible measures on how consumption growth reacts to predictable income changes. These statistics are computed from an individual time series of 200 periods; the table presents averages over 20,000 consumers.[20] We also report the nondurable to durable consumption ratio, average normalized wealth (defined as the sum of normalized financial and physical assets), the percentage of voluntary equity over total wealth, and the proportion of time individuals individuals carry no voluntary equity.

4.1.1. No adjustment costs

First, note that with θ equal to the user cost of the durable—roughly 10% for our calibration—the constraint does not affect the intra-temporal allocation between the durable and the nondurable. The agent spends fixed proportions of cash-on-hand on both goods and consequently all reported statistics are identical for durables and nondurables. Second, observe that as the down payment increases, the average nondurable to durable ratio increases since buying the durable good is more costly in terms of current liquidity. Also, the agent carries more total wealth but less voluntary equity—the durable is liquid and can be sold to free required equity without cost when necessary. In fact, for down payments higher than 30%, consumers hold no voluntary equity.

With respect to nondurable consumption, we observe that both consumption growth and its volatility decrease monotonically with increasing down payments. Moreover, agents can smooth nondurable consumption considerably (the smoothness coefficient is well below 60% for down payments of 20% or higher). Durable consumption growth is also smoother than income. However, the volatility of durable consumption growth is nonmonotonic in θ.

There are two channels which explain the decrease in the volatility of nondurable consumption growth with increasing down payments. The first channel operates through the increase in total wealth: more wealth means agents have more resources available to smooth transitory income shocks. The second channel relates to the fact that when down payments are high, agents spread out the accumulation of required wealth holdings in response to permanent income shocks. For example, if there is an above-average permanent income shock, the agent would like to increase consumption (durable and nondurable). A higher level of durable consumption, however, requires more wealth holdings in the form of the required down payment. Instead of increasing required wealth

[20] We simulate 250 periods but we ignore the first 50 to insulate the results from the influence of initial conditions. Normalized cash-on-hand converges to a stationary distribution quickly, about 12–15 periods starting from zero assets.

Table 1
Microeconomic results from individual time series

θ	0.0	0.05	~0.1[a]	0.2	0.3	0.4	0.5	1.0
Panel A. No adjustment costs								
Avg. g_C (%)	2.49	2.45	2.38	2.27	2.21	2.17	2.15	2.11
g_C smoothness	0.88	0.83	0.76	0.63	0.54	0.47	0.43	0.36
	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
g_C sensitivity	−0.17*	−0.17*	−0.14*	−0.09	−0.04	−0.01	0.00	0.01
	(0.06)	(0.06)	(0.05)	(0.05)	(0.04)	(0.03)	(0.03)	(0.03)
Avg. g_K (%)	2.22	2.29	2.38	2.45	2.39	2.29	2.23	2.11
g_K smoothness	0.56	0.65	0.76	0.84	0.78	0.66	0.57	0.35
	(0.02)	(0.02)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
g_K sensitivity	−0.05	−0.11*	−0.14*	−0.14*	−0.08	−0.02	0.01	0.03
	(0.04)	(0.05)	(0.05)	(0.06)	(0.06)	(0.05)	(0.04)	(0.03)
C-K ratio	0.35	0.35	0.36	0.37	0.38	0.39	0.40	0.45
Avg. wealth	0.03	0.13	0.24	0.43	0.63	0.82	1.00	1.82
Avg. q (%)	100.0	17.4	7.2	1.7	0.2	0.0	0.0	0.0
$q = 0$ (%)	40.9	43.1	52.2	72.7	91.1	98.5	99.8	100.0
Panel B. Adjustment costs								
Avg. g_C (%)	2.46	2.44	2.43	2.39	2.34	2.32	2.29	2.25
g_C smoothness	0.87	0.85	0.83	0.78	0.73	0.71	0.67	0.60
	(0.04)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.04)
g_C sensitivity	−0.16*	−0.16*	−0.14*	−0.08	−0.03	0.02	0.04	0.07
	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)	(0.05)	(0.05)	(0.04)
Avg. g_K (%)	5.02	5.00	4.76	4.71	4.52	4.28	4.07	3.35
g_K smoothness	2.65	2.65	2.45	2.42	2.31	2.16	2.04	1.57
	(0.14)	(0.14)	(0.14)	(0.14)	(0.13)	(0.12)	(0.11)	(0.08)
g_K sensitivity	−0.03	0.03	0.06	0.12	0.12	0.11	0.08	0.10
	(0.19)	(0.19)	(0.17)	(0.17)	(0.16)	(0.15)	(0.15)	(0.11)
C-K ratio	0.37	0.38	0.39	0.40	0.41	0.43	0.44	0.49
Avg. wealth	0.03	0.13	0.24	0.45	0.66	0.86	1.05	1.86
Avg. q (%)	100.0	20.7	12.9	10.9	10.9	10.9	10.9	9.7
$q = 0$ (%)	41.7	42.8	43.5	40.9	38.0	34.8	31.8	30.7

Notes: g_C and g_K are the growth rate of the nondurable and the durable, respectively. The rows labelled 'smoothness' report the ratio of the standard deviation of consumption growth to that of income growth. The rows labelled 'sensitivity' report the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. The row labelled 'avg. wealth' presents normalized wealth, the sum of financial wealth plus durable wealth divided by permanent income, $(a + k)$. q is normalized voluntary equity, the wealth held in excess of the required down payment. The row labelled 'avg. q (%)' is the percentage of voluntary equity on total wealth. In all cases, $R = 1.02$, $\psi = 0.915$, $\varphi = 0.795$, $\beta = 1/1.05$ and $\rho = 2$. $\phi = 0.05$ in the adjustment cost case. The parameters for the different income shocks are: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_T = 0$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. The implied growth rate for labor income is 2.63% and its standard deviation 11.07%. The statistics reported are calculated from an individual time series of 200 periods; the table shows averages over 20,000 individuals. The standard deviation of the smoothness ratio across the 20,000 individuals, as well as the average standard error of the regression coefficient of the excess sensitivity parameter, are in parentheses.

[a]Down payment equal to the user cost.

*Significant at the 5% level.

Table 2
Permanent versus transitory shocks. Individual time series. No adjustment costs

θ	0.0	0.05	~0.1[a]	0.2	0.3	0.4	0.5	1.0
Panel 1. Permanent shocks only								
g_C smoothness	1.33	1.15	1.00	0.87	0.81	0.77	0.75	0.68
	(0.02)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
g_C sensitivity	−0.30*	−0.15	0.00	0.10	0.12*	0.11*	0.11*	0.08
	(0.09)	(0.08)	(0.07)	(0.06)	(0.06)	(0.05)	(0.05)	(0.05)
g_K smoothness	1.00	1.07	1.00	0.79	0.65	0.57	0.51	0.37
	(0.00)	(0.01)	(0.00)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)
g_K sensitivity	0.00	−0.07	0.00	0.17*	0.24*	0.25*	0.24*	0.18*
	(0.07)	(0.08)	(0.07)	(0.06)	(0.04)	(0.04)	(0.03)	(0.02)
Panel 2. Transitory shocks only								
g_C smoothness	0.80	0.77	0.69	0.54	0.42	0.32	0.27	0.15
	(0.04)	(0.04)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)
g_C sensitivity	−0.21*	−0.22*	−0.20*	−0.14*	−0.09*	−0.05*	−0.03	−0.01
	(0.05)	(0.05)	(0.05)	(0.04)	(0.03)	(0.02)	(0.02)	(0.01)
g_K smoothness	0.25	0.47	0.69	0.86	0.82	0.69	0.59	0.35
	(0.01)	(0.01)	(0.03)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
g_K sensitivity	−0.06*	−0.14*	−0.20*	−0.23*	−0.17*	−0.10*	−0.06	−0.01
	(0.02)	(0.03)	(0.05)	(0.06)	(0.06)	(0.05)	(0.04)	(0.02)

Notes: All definitions and parameters as in Table 1 except in Panel 1 $\sigma_v = 0$, and in Panel 2 $\sigma_N = \sigma_G = 0$.
[a]Down payment equal to the user cost.
*Significant at the 5% level.

immediately to its new desired level, the agent accumulates the new down payment requirement over time so that nondurable consumption does not suffer temporarily. As θ increases, the burden imposed by the down payment rises resulting in accumulation of the required down payment over a longer period of time, and thus, smoother consumption growth.

In the event where agents were only subject to permanent shocks, the degree of consumption smoothing would be in fact controlled by the relative magnitude of the down payment and the user cost. When the constraint binds, the short-run cost of the durable is effectively θ, the cost of a unit of durable consumption in terms of foregone nondurable consumption. On the other hand, the user cost can be thought of as the durable's long-run price in terms of the agent's inter-temporal budget constraint. If the two prices are equal and the agent is constrained, the wealth requirement imposes a cost (in terms of foregone nondurable consumption) that is the same as the user cost. This sends a signal to the agent to adjust fully to the new desired level of durable consumption. When θ is higher than the user cost, the agent gradually adjusts to the new desired level of durable consumption over several periods; a full adjustment would impose too great a sacrifice of nondurable consumption relative to the unconstrained situation. When θ is lower than the user cost, the agent faces favorable credit conditions and consumption growth overshoots income growth. Panel 1 in Table 2, which presents a simulation with only permanent shocks, illustrates the point.

The nonmonotonicity in durable volatility originates in the durable's role as a store of wealth (compare panel 1 to panel 2 in Table 2). When an agent faces a transitory negative income shock, after voluntary equity runs out, required equity holdings are liquidated and used to smooth nondurable consumption. As θ increases, the agent carries less voluntary equity resulting in more occasions when durable consumption is reduced to convert required wealth into nondurable consumption. One may expect, then, to observe less smoothing of the durable as θ increases. However, this is not the case for the entire range of θ. Eventually, durable consumption growth becomes smoother as the down payment increases. This is because for high values of θ, the amount of forced saving is so high that the efficiency of transforming required wealth holdings into nondurable consumption increases and requires a less dramatic reduction in durable consumption. Since the wealth requirement is proportional to the durable stock, doubling the wealth requirement implies halving the amount of durable reduction necessary to yield a given amount of liquid resources. As a consequence, the volatility of durable consumption growth ultimately falls as the down payment requirement rises.

Returning to Table 1, we can see that excess smoothness of consumption is not always associated to excess sensitivity at the individual level. The reason is that this simple test for excess sensitivity fails to separate the effects of transitory and permanent shocks. Note that the excess sensitivity coefficient is negative for low values of θ but becomes insignificant for higher down payments. The liquidity constraint in combination with our specification for individual income growth—an MA(1)—implies a negative correlation between current consumption growth and lagged income growth: a low transitory innovation now signals high income growth next period but if the agent is constrained consumption cannot fully adjust in the current period. As θ increases, agents accumulate more wealth, they are constrained less often and the correlation weakens. Moreover, for down payments higher than the user cost, the sluggish response of consumption to permanent income shocks generates a positive correlation between lagged income growth and current consumption growth (and the opposite for down payments lower than the user cost). For high enough down payments, the effects cancel out resulting in no excess sensitivity. In Table 2, we isolate the effects of transitory and permanent shocks and these patterns can be seen more clearly. Briefly, the model produces excess sensitivity to transitory income shocks for low down payments (since agents are constrained more often), and to permanent income shocks if the down payment requirement is sufficiently above (or below) the user cost due to the slow adjustment of consumption.

The empirical evidence on excess sensitivity in microeconomic data is mixed, in large part due to the difficulty in isolating the predictable component of income in real data. Hall and Mishkin (1982) find a negative correlation between consumption growth and lagged income growth while Attanasio and Weber (1995) argue that after accounting for changes in household composition and labor supply there is no evidence of excess sensitivity in U.S. microdata. In general, studies that exploit exogenous predictable changes in household income (instead of using lagged information) find small but significant excess sensitivity coefficients, particularly for liquidity constrained consumers (e.g., Shea, 1995; Parker, 1999; Souleles, 1999). This is consistent with our results.

4.1.2. *Adjustment costs*

With adjustment costs, things are slightly more complicated. Agents change the durable stock infrequently in order to minimize the transaction cost. Fig. 3 depicts a simulation in which an individual receives average income shocks for a number of periods using a down payment of 30%. The graph shows normalized durable consumption, nondurable consumption, and voluntary equity. In the no adjustment cost case, the agent keeps c

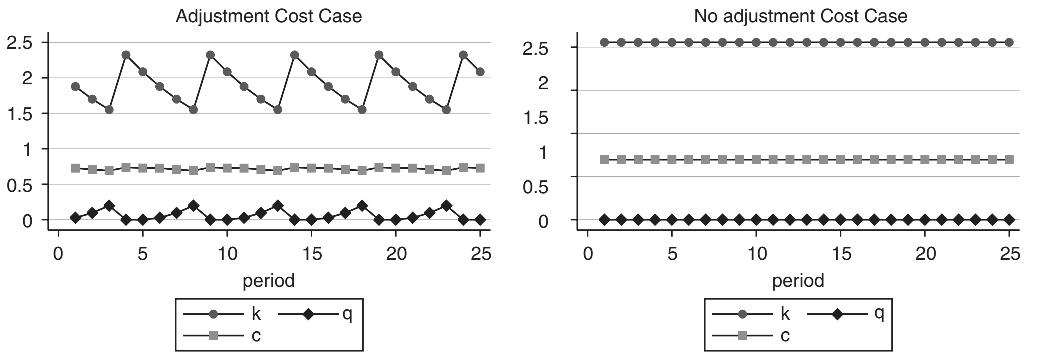

Fig. 3. Consumption and voluntary equity paths with average shocks. Thirty percent down payment.

and k constant and carries no voluntary equity. In the adjustment cost case, k follows an (S,s) rule. Even small adjustments are costly, so agents let durables depreciate until the s trigger is reached and adjustment takes place. Note that agents build up voluntary equity close to the period of adjustment, and that nondurable consumption, while relatively smooth, suffers slightly at the time of adjustment of the durable.

Table 1 shows that, for a given θ, the C-K ratio is higher with adjustment costs since the durable good becomes less attractive. Also, while wealth increases with the down payment, just as in the no adjustment cost case, voluntary equity does not go to zero; in fact it increases with the down payment in absolute value. It is now costly to sell the durable to free required equity so agents try to avoid this situation by keeping a small liquid buffer stock to deal with negative transitory shocks. Furthermore, for down payments above the user cost, agents are constrained less often with adjustment costs. For example, for $\theta = 1$, q is always zero without adjustment, but only 30.7% of the time with adjustment costs. The counterpart is that the liquidity constraint binds more tightly at the time of adjustment.

Unlike the no adjustment cost case, durable consumption growth is now more volatile than income growth. This is due to the optimal (S,s) adjustment rule for the durable. In the inaction region, the durable growth rate is equal to minus the depreciation rate, while during the year of adjustment durable consumption growth is quite substantial (see Fig. 4).

Also, now the durable becomes smoother with increasing down payments. This is because the liquidity constraint affects the size of the (S,s) bands. Fig. 5A illustrates this point. It depicts a simulation in which an individual receives average income shocks using two different down payments: 10% and 30%. We observe that increasing θ narrows the (S,s) bands resulting in smoother durable consumption growth. Nondurable consumption growth volatility also decreases with increasing down payments, in this case because of the extra wealth (voluntary and required equity) the agent accumulates for higher down payments.[21]

[21]Fig. 5A, shows that in the absence of shocks, nondurable consumption is slightly more volatile for the higher down payment because the agent needs to further increase voluntary equity close to the adjustment period. The extra equity is useful when dealing with transitory shocks, which results in a lower volatility of the nondurable when shocks are present.

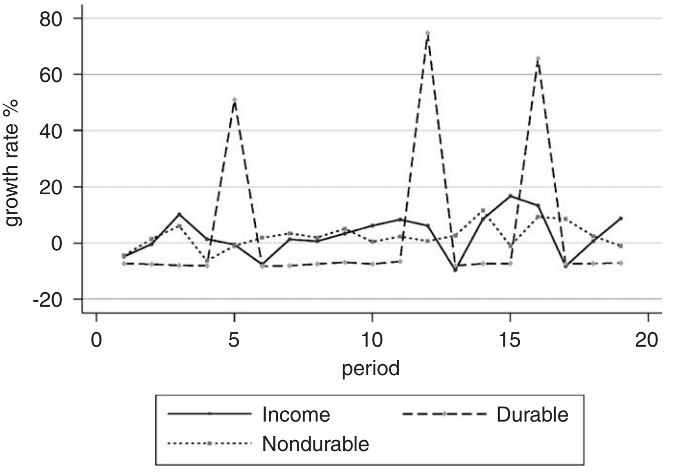

Fig. 4. Individual consumption and income growth rates with adjustment costs. Thirty percent down payment.

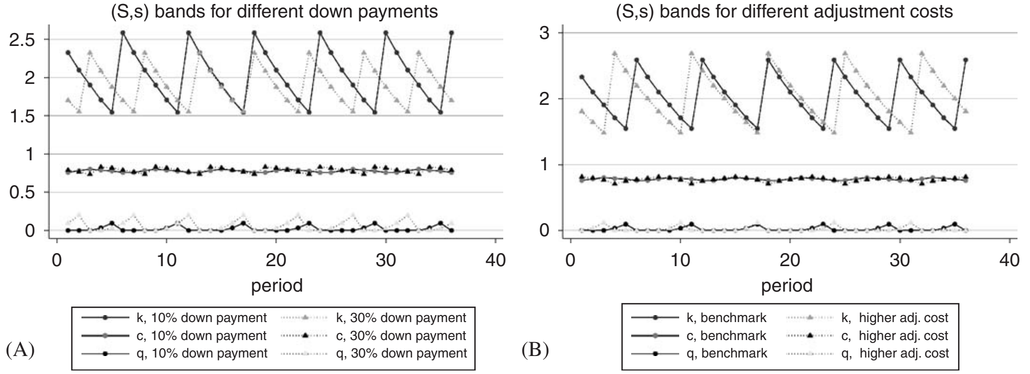

Fig. 5. Adjustment to permanent shocks.

Finally, note that nondurable consumption growth is still smoother than income growth, but smoothing is slightly less effective than in the no adjustment cost case. This is because it is now more expensive to use the durable stock to smooth nondurable consumption. Agents carry more voluntary equity to compensate but on occasion they must liquidate the durable, which is costly. Overall, smoothing is less effective. In fact, the higher the adjustment cost, the more volatile nondurable and durable consumption growth become for a given down payment (compare Table 3 with adjustment cost $\phi = 0.1$ to the benchmark results for $\phi = 0.05$ in Table 1). The mechanism behind the result is illustrated in Fig. 5B. The figure presents a simulation in which an individual receives average income shocks using two different adjustment costs, 5 and 10%, but the same down payment, 10%. For the durable, the adjustment cost widens the (*S*,*s*) bands—the durable is held for a longer period to minimize payment of adjustment costs—resulting in higher volatility. For the nondurable, the increased illiquidity of the durable means poorer smoothing.

Table 3
Higher adjustment costs. Individual time series

θ	0.0	0.05	$\sim$0.1[a]	0.2	0.3	0.4	0.5	1.0
g_C smoothness	0.87	0.86	0.84	0.82	0.80	0.78	0.78	0.76
	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.05)	(0.05)	(0.05)
g_K smoothness	3.21	3.20	3.20	2.94	2.88	2.78	2.62	1.98
	(0.17)	(0.18)	(0.18)	(0.17)	(0.16)	(0.15)	(0.14)	(0.11)
Avg. wealth	0.03	0.14	0.25	0.45	0.66	0.86	1.06	1.86
Avg. q (%)	100.0	25.7	17.4	14.7	14.5	14.5	15.0	13.4
$q = 0$ (%)	41.4	42.2	42.3	38.4	35.2	31.4	28.2	24.1

Notes: All definitions and parameters as in Table 1 except for the adjustment cost parameter, ϕ, which is 0.1 in this case.
[a]Down payment equal to the user cost.

Table 4
Stylized facts in U.S. aggregate data. 1959–2001

	Excess smoothness	Excess sensitivity
C		
Nondurables + services	0.60	0.20
		(2.15)
Nondurables + services − housing	0.67	0.16
		(1.85)
Nondurables + services − shoes and clothing	0.60	0.21
		(2.30)
Nondurables + services − housing − shoes and clothing	0.67	0.18
		(1.71)
K		
Non-residential durable stock	0.98	0.66
		(5.78)
Residential durable stock	0.36	0.19
		(3.78)
Non-residential + residential stock	0.42	0.26
		(4.93)

Notes: *Excess smoothness* is measured as the ratio of the standard deviation of consumption growth to the standard deviation of the income growth. *Excess sensitivity* refers to the OLS coefficient from a regression of consumption growth on lagged income growth and a constant (t-statistic in parentheses). All flow series are from the *National Product and Income Accounts* and all stock series from *Fixed Assets and Consumer Durable Goods Accounts* published by the Bureau of Economic Analysis. All series are per capita and appropriately deflated.

4.2. Aggregate consumption

The main question in this paper is whether our model can account for the macroeconomic stylized facts of excess sensitivity and excess smoothness of nondurable consumption. We now explore the aggregate implications of the model. We present the relevant statistics for U.S. aggregate data in Table 4, and then compare them to the numbers generated by our model reported in Table 5.

Table 5
Macroeconomic results from aggregate time series

θ	0.0	0.05	~0.1[a]	0.2	0.3	0.4	0.5	1.0
Panel A. No adjustment costs								
Avg. g_C (%)	2.18	2.17	2.17	2.16	2.16	2.16	2.16	2.15
g_C smoothness	1.16	1.07	0.99	0.87	0.80	0.76	0.73	0.65
	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
g_C sensitivity	−0.12	−0.06	0.00	0.08	0.11*	0.11*	0.11*	0.08
	(0.08)	(0.08)	(0.02)	(0.06)	(0.06)	(0.05)	(0.05)	(0.05)
Avg. g_K (%)	2.17	2.17	2.17	2.16	2.15	2.15	2.15	2.14
g_K smoothness	1.04	1.04	0.99	0.82	0.66	0.56	0.50	0.35
	(0.01)	(0.01)	(0.00)	(0.01)	(0.02)	(0.03)	(0.03)	(0.03)
g_K sensitivity	−0.03	−0.04	0.00	0.12*	0.21*	0.24*	0.23*	0.17*
	(0.07)	(0.07)	(0.02)	(0.06)	(0.05)	(0.04)	(0.03)	(0.02)
Avg. wealth	0.03	0.13	0.24	0.44	0.65	0.87	1.09	2.18
Avg. q (%)	100.0	17.6	7.2	1.6	0.2	0.0	0.0	0.0
φ	0.74	0.77	0.80	0.84	0.89	0.94	0.89	1.25
Panel B. Adjustment costs								
Avg. g_C (%)	2.17	2.17	2.16	2.16	2.15	2.15	2.15	2.15
g_C smoothness	1.06	0.99	0.91	0.78	0.67	0.60	0.56	0.47
	(0.007)	(0.01)	(0.01)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)
g_C sensitivity	−0.06	−0.03	0.01	0.09	0.16*	0.20*	0.20*	0.16*
	(0.075)	(0.07)	(0.18)	(0.06)	(0.05)	(0.04)	(0.04)	(0.04)
Avg. g_K (%)	2.17	2.17	2.17	2.16	2.16	2.16	2.15	2.15
g_K smoothness	1.05	1.00	0.94	0.86	0.80	0.74	0.67	0.48
	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.03)
g_K sensitivity	0.09	0.15*	0.21*	0.33*	0.38*	0.41*	0.36*	0.30*
	(0.07)	(0.07)	(0.07)	(0.06)	(0.05)	(0.04)	(0.04)	(0.03)
Avg. wealth	0.03	0.13	0.25	0.48	0.72	0.96	1.20	2.34
Avg. q (%)	100.0	19.7	12.8	10.8	10.8	10.8	10.8	9.1
φ	0.9	0.95	1	1.1	1.2	1.25	1.3	1.6

Notes: g_C and g_K are the growth rate of the nondurable consumption and the durable, respectively. The rows labelled 'smoothness' report the ratio of the standard deviation of consumption growth to that of income growth. The rows labelled 'sensitivity' report the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. The row labelled 'avg. wealth' presents normalized wealth, the sum of financial wealth plus durable wealth divided by permanent income, $(a + k)$. q is normalized voluntary equity, the wealth held in excess of the required down payment. The row labelled 'avg. q (%)' is the percentage of voluntary equity on total wealth. In all cases, $R = 1.02$, $\psi = 0.915$, $\beta = 1/1.05$ and $\rho = 2$. $\phi = 0.05$ in the adjustment cost case. The parameters for the different income shocks are: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_T = 0$, $\sigma_N = 0.05$, and $\sigma_T = 0.07$. These imply a growth rate for income of 2.17% and a standard deviation of 2.5%. φ, the preference parameter for durable goods, is allowed to vary with θ to keep the C-K ratio constant and equal to 0.36. The statistics reported are calculated from an aggregate time series of 200 periods. Aggregate consumption and income are calculated as averages over 2,000 individuals. We report average statistics for 100 independent simulations. In parentheses, we present the standard deviation of the smoothness ratio across the 100 simulations, as well as the average standard error of the regression coefficient of the excess sensitivity parameter.

[a]Down payment equal to the user cost.

*Significant at the 5% level.

4.2.1. The aggregate excesses in the data

We obtain annual U.S. aggregate data from the BEA for the period 1959–2001. We use the following variables: disposable labor income, consumption expenditure on nondurables and services (with and without housing services and expenditure on clothing and shoes), and the stock of nonresidential durables and residences. Disposable labor income is calculated from several NIPA components.[22] All consumption series are per capita and deflated by their corresponding chain-type price deflator. Income is deflated by the personal consumption expenditure deflator.

Table 4 shows the excesses coefficients for each consumption series.[23] Note that all consumption variables exhibit smoothness relative to income except for the nonresidential durable stock. Moreover, they all exhibit excess sensitivity. In the data, K is smoother than C and the excess sensitivity coefficient is higher for K than for C. The aggregates that best match our model are nondurable expenditure plus services minus housing services for C, and the residential stock plus the nonresidential durable stock for K.[24] For these aggregates, the excess sensitivity coefficients are 0.16 for the nondurable and 0.26 for the durable. The excess smoothness ratios are 0.67 for the nondurable and 0.42 for the durable.

4.2.2. The aggregate excesses in the model

Can our model reproduce these numbers? In order to determine the macroeconomic implications of the model, we explicitly aggregate over consumers. We generate idiosyncratic and aggregate labor income shocks from the assumed income distributions for 2,000 consumers during 200 periods. Given the calculated individual optimal consumption rules, we use the generated shocks to simulate nondurable and durable consumption for each consumer. Aggregate consumption and aggregate income in a given period are calculated as averages across individuals.[25] Then, the relevant statistics (i.e., the excess smoothness ratio, etc.) are calculated using the aggregate time series. The procedure is repeated for 100 independent simulations. Table 5 presents averages across the 100 simulations of the relevant statistics.

We start with the no adjustment cost case. With aggregation, idiosyncratic shocks cancel out and what remains is the influence from the common shock, which is permanent. Not surprisingly then, when the down payment parameter is equal to the user cost, the model does not deliver either excess smoothness or excess sensitivity and the results are similar to those of the standard buffer-stock model. Ludvigson and Michaelides (2001), Table 2, report an excess sensitivity coefficient of 0.001 and an excess smoothness value of 0.99 for our particular calibration, identical to ours. Moreover, when the down payment is lower than the user cost the model generates excess volatility. For down payments higher than the user cost, we obtain robust excess smoothness and in some cases excess sensitivity. As

[22]Specifically, disposable labor income is the sum of wages and salaries plus other labor income, minus personal contributions for social security and taxes. Taxes are defined as the fraction of wage and salary income in total income, times personal tax and nontax payments.

[23]We acknowledge that referring to the excess sensitivity and excess smoothness of the durable is an abuse of terminology, as these terms refer to the empirical evidence for the nondurable. We report analogous coefficients for the durable in order to provide an initial guideline of the fit of our model to the durable as well.

[24]In our specification, services from housing are derived from K, not from C.

[25]We simulate 250 periods but the first 50 periods are discarded to insulate results from initial conditions. Using more than 2,000 consumers does not change the results.

discussed in Section 4.1, agents choose not to adjust consumption levels immediately when facing permanent income shocks if down payments are burdensome, preferring to spread out the accumulation of required wealth holdings. The higher the down payment, the longer the adjustment process. Then, excess sensitivity appears because consumption optimally responds with a lag to changes in income. In other words, excess smoothness and excess sensitivity are the same phenomenon here. Of course, there are alternative explanations for aggregate excess sensitivity which our model does not incorporate such as time aggregation or household production (see Christiano et al., 1991; Baxter and Jermann, 1999). Note that as in the data, in our simulations K is smoother than C and its excess sensitivity coefficient is higher.

Table 5 shows that excess smoothness rises monotonically with increases in the down payment, but excess sensitivity is nonmonotonic. How is this possible? The answer lies in the simple test for excess sensitivity that we are using. Let us simulate a representative individual who receives the aggregate income process.[26] Let him receive average income shocks all periods except period 0. Fig. 6 depicts the paths for the growth rates of nondurable and durable consumption for three different down payments: 10%, 30% and 100%. Durable and nondurable consumption growth rates do not fully react to the change in income for down payments higher than the user cost value, 10%, resulting in excess smoothness. The excess sensitivity coefficient calculated in our simulations measures the reaction of current consumption changes to changes in income in the previous period only. Consider period 1 in the graph. Durable consumption growth is higher for $\theta = 0.3$ than for $\theta = 1$, so we obtain a lower excess sensitivity coefficient for the higher down payment in spite of consumption growth being smoother because of the longer adjustment period. Including more lags in the specification would take care of the problem.

Burdensome down payments can then qualitatively explain the existence of excess sensitivity and excess smoothness of nondurable consumption. Quantitatively, for $\theta = 0.3$ the smoothness ratio is 0.8 and the sensitivity coefficient 0.11. While these numbers are still far from their empirical counterparts, they outperform the role of incomplete information—Ludvigson and Michaelides (2001), table 3, report an excess sensitivity coefficient of 0.104 and an excess smoothness ratio of 0.92.

When adjustment costs are introduced, the (S, s) rule used by agents to adjust durable stocks implies that, unlike the no adjustment cost case, consumption of an individual who receives aggregate income shocks may not exhibit the same properties as aggregate consumption. This is because agents adjust their durable stocks at different times (as they reach their specific triggers) when reacting to a common permanent shock. Thus, income shocks have a longer lasting effect on aggregate consumption. As a result, nondurable consumption is smoother in the adjustment cost case. In fact, for a down payment of 30%, the model can reproduce the actual excess sensitivity and smoothness observed in the data for the nondurable. The excess sensitivity coefficient in this case is 0.16, and the excess smoothness coefficient 0.67.

Furthermore, with adjustment costs, the durable also exhibits excess sensitivity and is smoother than income for down payments higher than the user cost, which was not the case at the individual level. At the aggregate level, durable consumption growth is

[26]With no adjustment costs, the results from our explicit aggregation and from an exercise that considers a representative agent who receives the aggregate income process are very similar. We do not tabulate the experiment here for brevity.

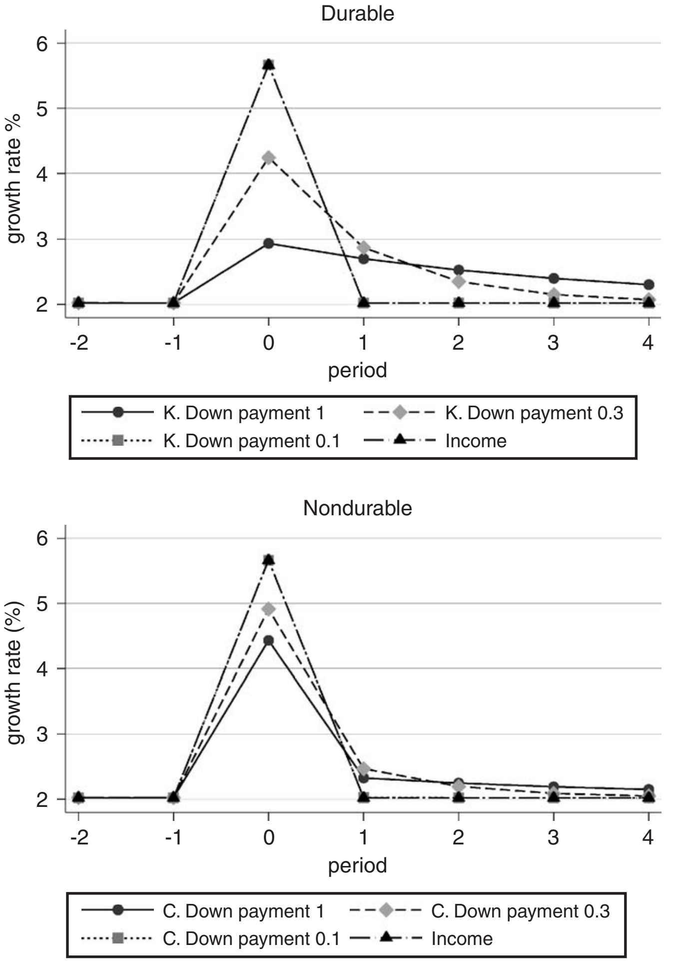

Fig. 6. Explaining smoothness and sensitivity.

smoother than income growth because different agents respond to the common permanent shocks at different times. In our baseline calibration, the durable is more volatile than the nondurable, unlike what we observe in the data. However, this result is not general. For example, when the transaction cost is 10% (see Table 6; $\phi = 0.1$), aggregate durable consumption growth is less volatile than nondurable consumption growth, in line with the data. This is because the higher the transaction cost, the less frequently consumers adjust their durable holdings (and the fewer consumers reach their triggers at the same time), resulting in a smaller reaction of durable consumption growth to current income growth.[27]

Table 6 presents some robustness analysis. We report excess smoothness and excess sensitivity coefficients for two down payments: 10% and 30%. The first two rows reproduce, for convenience, the results for the no adjustment cost case and the adjustment

[27]Note that nondurable consumption becomes slightly more volatile with the higher transaction cost, but it is still substantially smoother than in the no adjustment cost case.

Table 6
Further results from aggregate time series

		$\theta = 0.1$				$\theta = 0.3$			
		g_C		g_K		g_C		g_K	
		Ex. sen	Ex. sm.	Ex. sen	Ex. sm.	Ex. sen	Ex. sm.	Ex. sen	Ex. sm.
(1)	$\phi = 0$	0.00	0.99	0.00	0.99	0.11*	0.80	0.21*	0.66
		(0.02)	(0.00)	(0.02)	(0.00)	(0.06)	(0.01)	(0.04)	(0.02)
(2)	$\phi = 0.05$	0.01	0.91	0.21*	0.94	0.16*	0.67	0.38*	0.80
		(0.17)	(0.01)	(0.07)	(0.03)	(0.05)	(0.02)	(0.05)	(0.04)
(3)	$\phi = 0.1$	0.00	0.92	0.30*	0.82	0.10*	0.68	0.43*	0.66
		(0.07)	(0.01)	(0.05)	(0.04)	(0.05)	(0.02)	(0.04)	(0.05)
(4)	Higher ψ	0.04	0.87	0.30*	0.77	0.08	0.71	0.38*	0.61
		(0.06)	(0.01)	(0.05)	(0.03)	(0.05)	(0.02)	(0.03)	(0.04)
(5)	Buyer–seller	0.01	0.88	0.31*	0.87	0.11*	0.69	0.38*	0.67
		(0.06)	(0.01)	(0.06)	(0.04)	(0.05)	(0.02)	(0.04)	(0.04)
(6)	Seller > Buyer	0.10	0.82	0.19	0.91	0.14	0.69	0.27*	0.65
		(0.12)	(0.03)	(0.13)	(0.05)	(0.10)	(0.04)	(0.09)	(0.05)

Notes: g_C and g_K are the growth rate of nondurable consumption and durable consumption, respectively. All parameters as in Table 5, except the parameter being changed. For row (3), $\phi = 0.1$; for row (4), $\psi = 0.943$, which implies a depreciation rate of 5.7%; $\phi = 0.05$. For row (5), we use an alternative adjustment cost specification with the transaction cost divided between buyer and seller: $\zeta(K_t, K_{t-1}) = \phi_1 d\psi K_{t-1} + \phi_2 dK_t$, with $\phi_1 = \phi_2 = 0.025$. In row (6), the transaction cost is higher for negative investment than for positive investment (7.5% versus 2.5%).

*Significant at the 5% level.

cost case, respectively. The third row considers a higher adjustment cost (10%). The fourth row considers a lower depreciation rate for the durable, which implies a lower user cost. The last two rows summarize cases with slightly different adjustment cost specifications. In row (5), the transaction cost is divided between buyers and sellers (i.e., $\zeta(K_t, K_{t-1}) = \phi_1 d\psi K_{t-1} + \phi_2 dK_t,\ \phi_1 = \phi_2 = 0.025$). In row (6), the adjustment cost is paid by the seller, as in our original specification, but transaction costs are higher when investment is negative than when investment is positive (7.5% versus 2.5%). This is to allow for the possibility that some increases in the durable stock may be due to the purchase of extra units as opposed to the replacement of old ones.

In all cases, some excess smoothness is observed even for the lower down payment rate of 10%. For the 30% down payment, excess smoothness of the nondurable is similar to that observed in the data. Excess sensitivity is also the norm. The durable exhibits excess smoothness and excess sensitivity as well. Likewise, the durable is smoother than the nondurable good as in the data. However, to match the empirical figures for durables, higher down payments are necessary.

In summary, our model provides a plausible explanation for the results of excess sensitivity and excess smoothness of nondurable consumption in aggregate data. Consumption optimally responds with a lag to changes in income as consumers spread out the burden of down payments over several periods. With nonconvex adjustment costs, we find an even more prolonged adjustment in the aggregate resulting in higher excess sensitivity and excess smoothness for nondurable consumption growth. In order to quantitatively match the excesses, we need a high enough down payment relative to the user cost. While we do not argue that this is the only possible explanation for these so-called consumption excesses, we believe that it should not be overlooked.

5. Conclusions

This paper studies a buffer-stock model of saving where agents consume both durable and nondurable goods and durables serve as collateral. We consider variations of the model with and without adjustment costs in the durable market. We show that the constraint can alter the allocation of resources between the durable and the nondurable and has implications for the volatility of the two goods. Indeed, only when the down payment requirement exactly equals the user cost of the durable and there are no transaction costs should we study durables and nondurables separately.

We find that for an individual, nondurable consumption growth unambiguously becomes more volatile relative to income when the required down payment for purchases of durables in an economy is lowered. Moreover, this result is preserved by aggregation. For the durable, the results are slightly more complex, and while some nonmonotonicities exist at the individual level, aggregate durable consumption growth is also more volatile for lower down payments. For an individual, this result is explained in part by the fact that higher down payment requirements translate into higher wealth holdings to deal with negative transitory income shocks—since down payments act as a form of forced saving—and in part by the fact that consumers choose to gradually adjust their consumption when facing permanent income shocks in order to spread out the burden of the down payment. At the aggregate level, it is the latter effect that survives. Moreover, this gradualism or sluggish response of consumption to permanent income shocks generates robust excess smoothness and excess sensitivity in an explicit aggregation of the model for plausible parameter values. The model with adjustment costs can match the empirical evidence for nondurable consumption.

Another interesting implication of the model is that decreases in down payments and transaction costs reduce average wealth. For example, reducing the down payment from 30% to 20% decreases average wealth holdings by over 30% (see Table 5). For a down payment of 30%, eliminating the adjustment cost reduces average wealth holdings by 10%. Thus, differences in the durables market may play an important role in explaining differences in saving rates across countries or even the decrease in the saving rate in the U.S. over the last decades. This question, however, is better addressed in a general equilibrium setting.[28]

We stress that lower down payment requirements imply that households voluntarily lower wealth holdings (because of their impatience) and voluntarily accept the cost of greater volatility. In this model, ceteris paribus, consumers should be better-off in an economy with lower down payments.

Several factors not included in the model may further refine the results of this paper and deserve attention for future research. First, some consumers may simply give up saving for down payments if the cost is too prohibitive. An interesting extension to the current model

[28] In a related paper, Díaz and Luengo-Prado (2002) find that decreasing down payments or transaction costs lead to slightly higher real interest rates in general equilibrium. The increase in the interest rate (which increases the user cost) is not big enough to overturn the results in this paper.

would be to allow for this phenomenon which would require the explicit presence of a rental market. Second, the model would benefit from the inclusion of stochastic durable prices. Finally, the incorporation of transitory aggregate shocks could shed light onto the cyclical behavior of the consumption puzzles.

Appendix A. Numerical procedures

A.1. Euler equation iteration

A.1.1. Derivation of key equations (10) and (11)

Using our particular utility specification, we can write Eqs. (5) and (6), the Euler–Lagrange necessary conditions, as follows:

$$-C_t^{-\rho} + \beta R \mathrm{E}_t\{[C_{t+1}(X_{t+1})]^{-\rho}\} + \lambda_t = 0, \tag{5$'$}$$

$$\varphi K_t^{-\rho} - \beta(R-\psi)\mathrm{E}_t\{[C_{t+1}(X_{t+1})]^{-\rho}\} - \theta\lambda_t = 0. \tag{6$'$}$$

As in Carroll (1997), we rewrite the equations in ratio terms by taking advantage of the homogeneity of degree ρ of marginal utility and dividing all variables by permanent income:

$$-c_t^{-\rho} + \beta R \mathrm{E}_t\{(G_{t+1}N_{t+1})^{-\rho}[c_{t+1}(x_{t+1})]^{-\rho}\} + \lambda_t P_t^{\rho} = 0, \tag{5$'$}$$

$$\varphi k_t^{-\rho} - \beta(R-\psi)\mathrm{E}_t\{(G_{t+1}N_{t+1})^{-\rho}[c_{t+1}(x_{t+1})]^{-\rho}\} - \theta\lambda_t P_t^{\rho} = 0, \tag{6$'$}$$

where $x_{t+1} \equiv X_{t+1}/P_{t+1} = (G_{t+1}N_{t+1})^{-1}[R(x_t - c_t) + (\psi - R)k_t] + V_{t+1}$, $c_t \equiv C_t/P_t$ and $k_t \equiv K_t/P_t$.

When the agent is not liquidity constrained, $\lambda_t = 0$. Moreover, as we know from the intra-temporal condition, $c_t/k_t = \Omega$. Using these facts and the definition of x_{t+1} above, Eq. (5$'$) can be written as follows:

$$\beta R \mathrm{E}_t\left\{(G_{t+1}N_{t+1})^{-\rho}\left(c_{t+1}\left[(G_{t+1}N_{t+1})^{-1}\left(R(x_t - c_t) + \left(\frac{\psi - R}{\Omega}\right)c_t\right) + T_{t+1}\right]\right)^{-\rho}\right\} \\ - c_t^{-\rho} = 0. \tag{10}$$

When the agent is constrained, $x_t = c_t + \theta k_t$. Also, we can solve for $\lambda_t P_t^{\rho}$ in Eq. (5$'$). Substituting into Eq. (6$'$), we can write:

$$\beta[\psi - R(1-\theta)]\mathrm{E}_t\Big\{(G_{t+1}N_{t+1})^{-\rho}\Big(c_{t+1}\Big[(G_{t+1}N_{t+1})^{-1}[\psi - R(1-\theta)]\frac{x_t - c_t}{\theta} \\ + T_{t+1}\Big]\Big)^{-\rho}\Big\} - \theta c_t^{-\rho} + \varphi\left(\frac{x_t - c_t}{\theta}\right)^{-\rho} = 0. \tag{11}$$

A.1.2. About the technique

With the two equations above, we can find the optimal rule for normalized nondurable consumption as a function of the unique state variable, normalized cash-on-hand, x. We denote the optimal rule as $c(x)$. Euler equation iteration requires assuming a finite horizon, T, and recursively solving backwards from the last period of life. To apply the method successfully, we need to: (i) evaluate the expectation, (ii) select an appropriate terminal condition and (iii) find a criterion to check if the agent is constrained.

In order to evaluate the expectation, we avoid numerical integration by replacing the continuous G_t, N_t and T_t processes by 5-point discrete approximations as suggested by

Tauchen (1986). With regards to the terminal condition, we assume that, as in Deaton (1992), the value of total assets is zero at time T, $a_T + k_T = 0$. Then $c_T(x) = x$ (the agent spends all his cash-on-hand on the nondurable).

In period $T-1$, for a given value of x, we can numerically compute the value c_{T-1} that satisfies the appropriate equation: the first one if the agent is not constrained and the second one if he is. We do so for a grid of values of x and numerically approximate the optimal consumption rule $c_{T-1}(x)$ through interpolation between the points of the x grid (we use cubic spline interpolation). Once we have $c_{T-1}(x)$, the same grid of x values is used to compute $c_{T-2}(x)$. With $c_{T-2}(x)$, $c_{T-3}(x)$ is computed, and so on.

Note that there is a shortcut to verify if the agent is constrained for a given value of x. At each time iteration, find $x_t^*(\theta)$, the exact value of cash-on-hand for which the liquidity constraint just binds. This can be done by noticing that at this point, $x_t^*(\theta) = c_t(x_t^*(\theta)) + \theta k_t(x_t^*(\theta))$ and $c_t(x_t^*(\theta)) = \Omega k_t(x_t^*(\theta))$. This implies that $c_t(x_t^*(\theta)) = \Omega(\Omega+\theta)^{-1}x_t^*(\theta)$. Then, we can just solve Eq. (10) for $x_t^*(\theta)$. For all $x \leqslant x_t^*(\theta)$ the agent is constrained, and vice versa.

Once we have the optimal policy function for the nondurable, $c(x)$, the optimal policy function for the durable, $k(x)$, can be calculated by using the intra-temporal relationship between the two goods

$$k(x) = \begin{cases} \theta^{-1}[x - c(x)], & x \leqslant x^*(\theta), \\ \Omega^{-1}c(x), & x > x^*(\theta). \end{cases}$$

A.1.3. Convergence conditions

Two sufficient conditions for the individual Euler equations (5) and (6) to define a contraction mapping for $\{c(x), k(x)\}$ are the conditions in Theorem 1 of Deaton and Laroque (1992). In our case:

$$\beta R \mathrm{E}_t[(G_{t+1}N_{t+1})^{-\rho}] < 1, \tag{15}$$

$$\beta(R-\psi)\mathrm{E}_t[(G_{t+1}N_{t+1})^{-\rho}] < 1. \tag{16}$$

Using the fact that G_{t+1} and N_{t+1} are independent and taking logs in the above equations, we obtain:

$$\frac{1}{\rho}[\ln(\beta) + \ln(R)] + \frac{\rho}{2}(\sigma_G^2 + \sigma_N^2) < \mu_G, \tag{17}$$

$$\frac{1}{\rho}[\ln(\beta) + \ln(R-\psi)] + \frac{\rho}{2}(\sigma_G^2 + \sigma_N^2) < \mu_G. \tag{18}$$

Eq. (17) is the 'impatience' condition derived by Deaton (1991) with $\mu_N = 0$. This condition ensures that borrowing is part of the unconstrained plan. For Eq. (18) to be satisfied, $R > \psi$. Moreover, as long as $0 < R - \psi < 1$, condition (17) is stricter than condition (18). Briefly, for $0 < R - \psi < 1$, the standard impatience condition common to buffer-stock models guarantees convergence. For $R - \psi > 1$, convergence is guaranteed by condition (18). For $R < \psi$, convergence is not guaranteed.

A.2. Finite-state approximation

The technique consists of specifying a finite-state problem that approximates the continuous one we want to solve. We replace the continuous state variables, k and q, with the finite sets, $\mathscr{K} = \{k_1, ..., k_{N_k}\}$ and $\mathscr{Q} = \{q_1, ..., q_{N_q}\}$. Note that the problem has been conveniently formulated in such a way that the control variables are next period's states. The liquidity constraint is implemented by setting $q_1 = 0$ and $q_i > 0, \forall q_i \in \mathscr{Q}, i > 1$. To deal with adjustment cost, we set

$$d = \begin{cases} 0, & |k_t - (G_t N_t)^{-1} \psi k_{t-1}| \leqslant \kappa, \\ 1, & |k_t - (G_t N_t)^{-1} \psi k_{t-1}| > \kappa, \end{cases}$$

where $\kappa = (k_n - k_l)/(N_k - 1)$. The precision of our solution increases as κ falls. This 'work around' solution may have some economic significance. It may be possible for the agent to make small changes to his durable stock, such as repairs, which do not require significant adjustment costs. If this is the case, the numerical formulation described here would be most appropriate.

As with the previous technique, all components of the income process are discretized. N_G points for G, N_N points for N, and N_T points for the transitory shock T. We then use value function iteration, which is sped up with an acceleration technique, modified policy function iteration with S states as described in Judd (1997). Briefly,

(1) Choose an initial guess V^0. Let $V^\ell = V^0$.
(2) Calculate $U^{\ell+1} = \mathscr{U} V^\ell$. For each (q_i, k_j), the mapping $\mathscr{U}$ is defined as:

$$\begin{aligned} U^{\ell+1}_{i,j,m,n,o} = \max_{q^+, k^+; q^+ \geqslant 0} & \; U[(G_m N_n)^{-1}\{Rq_i + [\psi(1 - d\phi) - R(1 - \theta)]k_j\} \\ & - \theta k^+ + T_o - q^+, k^+] + V^\ell(q^+, k^+) \equiv \mathscr{U} V^\ell_{i,j}. \end{aligned}$$

(3) Let $W^0 = V^\ell$. For each (q_i, k_j) and $s = 1, \ldots, S$, calculate:

$$\begin{aligned} & W^{s+1}(q_i, k_j) \\ & = \beta \frac{1}{(N_G N_N N_T)} \left\{ \sum_{m=1}^{N_G} \sum_{n=1}^{N_N} \sum_{o=1}^{N_T} (G_m N_n)^{1-\rho} U[(G_m N_n)^{-1}\{Rq_i + [\psi(1 - d\phi) \right. \\ & \quad \left. - R(1 - \theta)]k_j\} - \theta U_k^{\ell+1} + T_o - U_q^{\ell+1}, U_k^{\ell+1}] + W^s[U_q^{\ell+1}, U_k^{\ell+1}] \right\}. \end{aligned}$$

Set $V^{\ell+1} = W^S$.
(4) Iterate until convergence.

Note that the selection of appropriate bounds for the sets $\mathscr{K}$ and $\mathscr{Q}$ is key for the successful application of the technique. See Farr and Luengo-Prado (1999) for more information about this method and a comparison to Euler equation iteration.

For the construction of Table 1, we set $N_G = N_N = N_V = 5$; $N_k = 150$, $k_1 = 0.01$ and $k_{N_k} = 2.5$; $N_q = 70$, $q_1 = 0$ and $q_{N_q} = 0.3$ for $\theta \in [0, 0.4]$; $N_q = 90$, $q_1 = 0$ and $q_{N_q} = 0.4$ for $\theta = 0.5$; $N_q = 110$, $q_1 = 0$ and $q_{N_q} = 0.5$ for $\theta = 1$.

Appendix B. Proofs of propositions

Proof of Proposition 1. We know from the intra-temporal first order condition of the problem that the ratio of nondurable to durable consumption equals Ω, regardless of the value of θ. Moreover, $x_t = c_t + \theta k_t + q_t$. Both conditions imply the results. $\square$

Proof of Proposition 2.

(A) If $\theta = 0$, $x_t = c_t + q_t$. When the agent is constrained, $q_t = 0$ so $c(x) = x$. $k(x)$ does not depend on x in this case since it does not impose any cost on current liquidity. Note that k provides utility today but decreases cash-on-hand tomorrow ($\psi < R$). Therefore, there is an optimal level of k while x is below $x^*(\theta)$. Given that $k(x) = \Omega^{-1}(x - q)$ for $x \geqslant x^*(\theta)$, $k(x) = \Omega^{-1}x^*(\theta)$ for $x < x^*(\theta)$.

(B) Since the agent is liquidity constrained, $x = c(x) + \theta k(x)$. Then, $1 = c'(x) + \theta k'(x)$, $0 = c'(x) + \theta k'(x)$, with $c'(x) > 0$ and $k'(x) > 0$, and $c(0) = k(0) = 0$. From the intra-temporal first order condition we know that:

$$\frac{c(x^*(\theta))}{k(x^*(\theta))} = \Omega \quad \text{and} \quad \frac{c(x)}{k(x)} < \Omega \quad \forall x < x^*(\theta).$$

The closer x to $x^*(\theta)$, the lower the shadow price of the constraint. Therefore, the c-k ratio increases with x. The only way this can happen is with $c'(x) > 0$ and $k'(x) < 0$. Hence $c(x)$ must be convex and $k(x)$ concave while the agent is constrained.

(C) We observe from the intra-temporal first order condition that in this case, the agent is able to keep the c-k ratio constant and equal to Ω. Moreover, $c(x) + \theta k(x) = x$. Both conditions imply the result.

(D) The proof is similar to (B). In this case, the c-k ratio is higher than Ω when the agent is constrained and decreases towards Ω as x approaches $x^*(\theta)$. The only way this can happen is with a concave $c(x)$ and a convex $k(x)$. $\square$

References

Abowd, J., Card, D., 1989. On the covariance structure of earnings and hours changes. Econometrica 57, 411–445.

Alessie, R., Michael, P.D., Guglielmo, W., 1997. Intertemporal consumption, durables and liquidity constraints: a cohort analysis. European Economic Review 41, 37–59.

Attanasio, O., Weber, G., 1995. Is consumption growth consistent with intertemporal optimization? Evidence from the consumer expenditure survey. Journal of Political Economy 103, 1121–1157.

Bacchetta, P., Gerlach, S., 1997. Consumption and credit constraints: international evidence. Journal of Monetary Economics 40, 207–238.

Baxter, M., Jermann, U.J., 1999. Household production and the excess sensitivity of consumption to current income. American Economic Review 89, 902–920.

Bernanke, B., 1984. Permanent income liquidity and expenditure on automobiles: evidence from panel data. Quarterly Journal of Economics 99, 587–614.

Browning, M., Crossley, T., 1997. Shocks stocks and socks: consumption smoothing and the replacement of durables during an unemployment spell. McMaster University, Mimeo.

Caballero, R.J., 1993. Durable goods: an explanation of their slow adjustment. Journal of Political Economy 101, 351–384.

Campbell, J.Y., Deaton, A., 1989. Why is consumption so smooth? Review of Economic Studies 56, 357–374.

Campbell, J.Y., Mankiw, G.N., 1987. Are output fluctuations transitory? Quarterly Journal of Economics 102, 857–880.

Carroll, C.D., 1997. Buffer-stock saving and the life cycle/permanent income hypothesis. Quarterly Journal of Economics 112, 1–55.
Carroll, C.D., Dunn, W.E., 1997. Unemployment expectations jumping (S, s) triggers and household balance sheets. In: Bernanke, B.S., Rotemberg, J.J. (Eds.), NBER Macroeconomics Annual, pp. 165–217.
Chah, E.Y., Ramey, V.A., Starr, R.M., 1995. Liquidity constraints and intertemporal consumer optimization: theory and evidence from durable goods. Journal of Money, Credit, and Banking 27, 272–287.
Christiano, J.H., Eichenbaurn, M., Marshall, D., 1991. The permanent income hypothesis revisited. Econometrica 59, 397–424.
Cochrane, J.H., 1988. How big is the random walk in GNP? Journal of Political Economy 96, 893–920.
Deaton, A., 1991. Saving and liquidity constraints. Econometrica 59, 1121–1248.
Deaton, A., 1992. Understanding Consumption. Oxford University Press, Oxford, UK.
Deaton, A., Laroque, G., 1992. On the behaviour of commodity prices. Review of Economic Studies 59, 1–23.
Denizer, C.A., Iyigun, M.F., Owen, A., 2002. Finance and Macroeconomic Volatility. Contributions to Macroeconomics, vol. 2, No. 1, Article 7.
Díaz, A., Luengo-Prado, M.J., 2002. Precautionary saving and wealth distribution with durable goods. Mimeo.
Eberly, J.C., 1994. Adjustment of consumers' durables stocks: evidence from automobile purchases. Journal of Political Economy 102, 403–436.
Farr, C.C., Luengo-Prado, M.J., 1999. Solving micro-consumption models with durables, nondurables and liquidity constraints. Mimeo.
Flavin, M., 1981. The adjustment of consumption to changing expectations about future income. Journal of Political Economy 89, 974–1009.
Friedman, M., 1957. A Theory of the Consumption Function. Princeton University Press, Princeton, NJ.
Grossman, S.J., Laroque, G., 1990. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. Econometrica 58, 25–51.
Hall, R.E., 1978. Stochastic implications of the life cycle-permanent income hypothesis: theory and evidence. Journal of Political Economy 86, 971–987.
Hall, R.E., Mishkin, F.S., 1982. The sensitivity of consumption to transitory income: estimates from panel data on households. Econometrica 50, 461–480.
Hansen, L.P., Sargent, T.J., 1981. A note on Wiener–Kolmogorov prediction formulas for rational expectations models. Economics Letters 8, 255–260.
Japelli, T., Pagano, M., 1989. Consumption and capital market imperfections: an international comparison. American Economic Review 79, 1088–1105.
Judd, K.L., 1997. Numerical Methods in Economics. The MIT Press, Cambridge, MA.
Kose, M.A., Prasad, E.S., Terrones, M.E., 2003. Financial integration and macroeconomic volatility. IMF Working Paper No. 03/50.
Lam, P., 1989. Irreversibility and consumer durables expenditures. Journal of Monetary Economics 23, 135–150.
Ludvigson, S., Michaelides, A., 2001. Can buffer stock saving explain the consumption excesses. American Economic Review 91, 631–647.
MaCurdy, T.E., 1982. The use of time-series processes to model the error structure of earnings in longitudinal data analysis. Journal of Econometrics 18, 83–114.
Modigliani, F., Brumberg, R., 1954. Utility analysis and the consumption function: an interpretation of cross-section data. In: Kurihara, K.K. (Ed.), Post-Keynesian Economics, pp. 388–436.
Parker, J.A., 1999. The reaction of household consumption to predictable changes in social security taxes. American Economic Review 89, 959–973.
Peersman, G., Pozzi, L., 2004. Determinants of consumption smoothing. Universiteit Gent. Working Paper 2004/231.
Pischke, J.-S., 1995. Individual income, incomplete information and aggregate consumption. Econometrica 63, 805–840.
Shea, J., 1995. Union contracts and the life cycle/permanent-income hypothesis. American Economic Review 81, 186–200.
Souleles, S.N., 1999. The response of household consumption to income tax refunds. American Economic Review 89, 947–958.
Tauchen, G., 1986. Finite State Markov chain approximations to univariate and vector autoregressions. Economics Letters 20, 177–181.